2020

23rd Annual High School Mathematical Contest in Modeling (HiMCM) Summary Sheet

Team Control Number: 10997

Problem Chosen: B

Summary

As global biodiversity continues to face an unprecedented crisis imposed by habitat loss and invasive species, endangered plant species conservation is crucial for restoring environmental equilibrium. Conservation managers nowadays are facing difficult decisions due to limited resources and numerous projects awaiting funding. In this problem, we are tasked with coming up with a priority order of funding for the given 48 plant conservation projects under the Florida Rare Plant Conservation Endowment (FRPCE) Board.

Before developing our model, we first identify the relevant objectives that the Board would want to meet in deciding the fundraising schedule. Firstly, they would want to **maximize expected net benefit**, which is represented by a weighted score incorporating benefit, cost, taxonomic uniqueness and feasibility of the selected projects. Since we expect the Endowment to have a relatively constant annual revenue, they would also prefer schedules with **minimal fluctuations in yearly spending**. To meet these objectives, we identify the common characteristics of imperiled plant species and interpret the factors involved in their conservation.

In our model development, we establish a feasibility decay function to model the increasing risk of extinction if conservation actions are not taken or delayed. This is accomplished by logistic population growth with Allee effect. Next, to determine which projects to prioritize, we assign a **Priority Index** to each project in each year, which takes into account the project's benefit, taxonomic uniqueness, feasibility of success, total cost, and duration. Priority Index is determined using the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) and Entropy Weight Method. Subsequently, we employ Dynamic Programming with Greedy Algorithm to obtain an initial schedule that maximizes the sum of Priority Index of selected projects. The schedule specifies the projects that the Board should prioritize to fund in each year, and displays the required yearly expenditure.

Genetic Algorithm is then used to optimize our initial schedule. We use a comprehensive **total score** as our objective function, which incorporates both the expected net benefit as well as the standard deviation of our proposed annual funding schedule. The algorithm performs crossover, mutation, and tournament selection to reach a schedule with the best total score. After iterating 300 generations, we obtained our optimal schedule. For a schedule with a maximum funding cap of $500,000, the use of Genetic Algorithm can reduce standard deviation in yearly spending from 109443 to 39215.

To suggest to the Board an optimal fundraising schedule that can minimize funds raised and achieve long-term and reliable funding, we choose **$500,000** as the funding cap. In case they want to complete all the projects, we also provide them with the schedule with a funding cap **$2,000,000**. Lastly, we present our **priority order of funding** in a color-coded table which specifies the starting year of each project.

Keywords: Conservation of Endangered Plant Species, Logistic Model with Allee Effect, TOPSIS Enhanced by Entropy Weight Method, Dynamic Programming with Greedy Solution, Genetic Algorithm

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1 Introduction

1.1 Background

Florida, a biodiversity hotspot, is home to 215 endemic plant species due to its unique geographical location and environment. However, residential, agricultural and commercial development has contributed to habitat degradation and loss, while invasive plant species have competed with endemic species and formed single-species environments[1]. These factors caused the plant extinction rate in Florida to skyrocket with more than 60 plant species federally listed as endangered or threatened[2].

In light of this, authorities have come up with protection measures to restore ecological balance. The US Endangered Species Act of 1973 requires that all threatened and endangered species to be covered by a specific recovery plan[3]. In Florida, the Fish & Wildlife Foundation of Florida garners more than $4 million annually for conservation actions[4] and has helped to conserve 8,000 acres of critical wildlife habitat[5]. The Foundation has also initiated the Florida Rare Plant Conservation Endowment (FRPCE), a trust fund to support imperiled plant species conservation projects.

Nevertheless, conserving imperiled species remains an uphill task. One reason is limited funding, which poses a conundrum for conservation organisations: should they prioritize investment in species whose survival will benefit the most or those facing impending extinction[6]? Given the complexity involved in resource allocation, species recovery requires strategic and forward-looking planning.

1.2 Problem Restatement

Because of the aforementioned difficulties in prioritizing conservation projects, we propose a solution for the FRPCE Board to make decisions on which projects to invest and how to prioritize funding efficiently.

Firstly, we need to identify desirable goals for funding conservation projects and establish criteria to evaluate our generated fundraising schedule (i.e. plan of annual expenditure with time).

Secondly, we are asked to do research on the common attributes of endangered plant species as well as conventional protection measures. We should relate the factors involved in the attributes and measures to the ones used in our model.

Using our defined objectives and factors, we should develop models and algorithms to obtain an optimal funding schedule which minimizes the fluctuations in yearly budget but still achieves a high expected net benefit score.

Lastly, we need to translate our funding schedule into a priority plan that recommends the order of investment for the recovery projects, under different annual funding restriction.

Based on our analysis, we develop a mathematical model to schedule the 48 recovery projects provided in the question. Firstly, we consider the impact of time on the feasibility of success of projects. Afterwards, to evaluate each project based on the data given, we develop a Priority Index, which is then used to generate an initial schedule using Dynamic Programming and Greedy Algorithm. Finally, we incorporate the Genetic Algorithm to obtain an optimal schedule which minimizes fluctuations in yearly spending.

2 Assumptions and Notations

2.1 Assumptions and Justification

Assumption: No other organizations are conserving the 48 endangered plant species in Florida.
Justification: According to the *Federal and State Endangered and Threatened Species Expenditures*[7], Fish and Wildlife Service(FWS) plays a major part in funding conservation and recovery of Florida's endangered plant species. For instance, in 2017, *Polygala smallii* and *Dicerandra christmanii* only received funding from FWS. Since FWS conserves species by various initiatives, it is justifiable to assume that FRPCE is the one providing funds for plant conservation.

Assumption: *Feasibility of success* decreases with time if a project is delayed (i.e. does not start from Year 1 of the schedule).
Justification: When a project does not receive timely funding, the lack of conservation management causes the species to continue to face crisis and thus to decline in population. By research[8], the more critically endangered a species is, the more likely a project will fail to save it from extinction. We will account for this trend in more detail in our model development.

Assumption: *Benefit* and *taxonomic uniqueness* do not vary with time.
Justification:
1. As we have already accounted for the change in the risk of extinction in *feasibility of success*, and the difficulty level of performing conservation actions is unlikely to change for a planned project, for the ease of our model development, we assume the *benefit* of a project to be constant.
2. The taxonomic information of a species is not likely to change over the course of the schedule (about 30 to 40 years) due to the modern rigorous biological classification system[9], so *taxonomic uniqueness* is also time-independent.

Assumption: The FRPCE Board has a relatively constant annual budget for conservation projects and provides funds for them on a yearly basis.
Justification: The Endowment uses investment income to fund conservation projects. Since no information is given, we can assume they generate a stable revenue. Additionally, in the U.S, conservation funds are usually issued annually[10]. The costs incurred by each project are given by year, so we assume that the FRPCE Board grants funding annually to each project.

Assumption: Some fluctuations in the annual budget are acceptable.
Justification: It is unrealistic to cap the annual funding at a precise amount. Hence, there should be some extent of fluctuation allowed when determining funding for each year.

2.2 Notations and Definitions

Symbol	Meaning	Units
b_i	Benefit of project i	-
u_i	Taxonomic Uniqueness of project i	-
c_i	Total Cost of project i	US dollar
$f_i(t)$	Feasibility of Success of project i in year t	-
N	Total number of years	year
ρ_i	Population density of species i	-
ρ_0	Critical population density	-
r	Extinction rate coefficient	-
$PRI_i(t)$	Priority Index of project i in year t	-
$T_k^{(i)}$	Starting year[1] of project i in schedule k	year
EB	Expected net benefit	-
TS_k	Total Score of schedule k	-

Table 1: Symbols used, their meaning and units

3 Model Development

3.1 Model Objectives

In deciding its funding priorities, the FRPCE Board has multiple desired objectives. These objectives are related to (but not limited to) the conservation status of a species, the existing fund as well as the proposed projects. To help them make their decisions, we identified and shortlisted some of the most important objectives and chose to include them in our modelling process.

1. **Achieve maximum total benefit.** This is the utmost priority for the Board since the endowment is set up to support conservation projects for Florida imperiled plant species. This total benefit should take into account benefit, taxonomic uniqueness as well as the feasibility of success for each project.

2. **Minimize fluctuations in yearly expenditure.** Since we do not have information on the Board's revenue each year, we assume them to be relatively constant. By minimizing fluctuations and thus keeping the annual budget to be relatively stable, the Board could balance funds available with accumulated spending.

3. **Achieve high cost-effectiveness.** For the fundraising plan to be reliable, the Board should aim to complete a large number, if not all, of the projects. The best plan should also keep the annual budget reasonably low.

4. **Fund chosen projects continuously until completion.** By providing long-term and reliable funding for conservation projects, the Board should ensure that each project receives adequate funds over the whole duration of the project.

The measures used to evaluate our generated funding plan will be established later in our model development section.

[1]In our model, we take Year 1 as the starting year of the schedule.

3.2 Explanation of Factors

3.2.1 Common Characteristics of Imperiled Plants

By research, endangered plant species have some common characteristics.

They usually have **specific requirements for the environment.** For instance, *Bonamia grandiflora*, an endangered species in Florida, is dependent on the sunny cleared areas left by periodic fires or physical disturbance. Fire suppression left their natural habitat overgrown and unsuitable for highly specialized scrub endemics that require open sunny patches. Thus, the loss of suitable habitat inevitably leads to the decline in the species' population. Some other species have requirements for periodic disruptions to flower or seed.

Some plant species are **naturally rare** and hence prone to extinction. They may have genetic self-incompatibility which prevents them from self-fertilization. The germination rate, reproductive rate is relatively low for the endangered species, possibly a result of their limited habitat.

Most of the listed species are **endemic** to Florida, which means that they are naturally present only in Florida. Most of them have a very small range size and are geographically concentrated. This causes the species to be more vulnerable to environmental or human disruptions, thus these species run a higher risk of extinction.

3.2.2 Protection Measures

From the *South Florida Multi-species Recovery Plan*[11], we identify key measures involved in these endangered species' protection.

On a **species-level**, to protect and enhance existing populations, research, monitoring, surveys are often needed, depending on the current availability of data.

On a **habitat-level**, major recovery actions include securing habitat through acquisition, conducting prescribed burns, controlling and eliminating invasive plant species, controlling access to areas where endangered plants are growing and (re)introducing plants to protected sites.

3.2.3 Factors Interpretation

From an environmental viewpoint, conservation of plant species aids in restoring ecological equilibrium and improving pollination, climate regulation, nutrition recycling, and carbon sequestration[12, 13]. Conservation of plant species also carries economic ripple effects like ecotourism. This is incorporated into the ***benefit*** factor which indicates the expected relative conservation value of funding one species over another. Besides, *benefit* of protection is directly related to the conservation status of each species, which specifies the level of endangerment.

The aforementioned measures to restore species' natural habitat require different amounts of money depending on the specific measures that need to be put in place. Specifically, some species' recovery plans involve land acquisition, landowner agreements and some require extensive research to be done. The ***total cost*** factor, c, takes this aspect of conservation into consideration.

Since there are uncertainties in plant species' reproduction and their response to some management actions like prescribed fires[2] is not studied thoroughly enough, the protection measures are not perfect solutions to save these species from extinction. The varying effectiveness of conservation management[14] and species' different vulnerabilities to environmental disruption are taken into account by the ***feasibility of success*** factor, $f(t)$.

The duration of each conservation project differs and is represented by the ***number of years*** factor, N.

The ***taxonomic uniqueness*** factor, u, indicates the relative rareness in genetics of a specific plant species and its phylogenetic diversity (evolutionary history stored in a species' genes).

3.3 Evaluating individual projects

3.3.1 Population Model

The logistic equation describes the growth rate of a population with limited resources. In order to predict the change in population size of the 48 imperiled plant species within the span of our schedule, we consider the logistic equation to describe population decrease. Typically, the logistic equation is:

$$\frac{d\rho_i}{dt} = -r\rho_i\left(1-\rho_i\right)$$

where ρ_i represents the population density of the ith species, t represents the time, r represents the growth coefficient. Here, population density means the ratio of current population to the region's carrying capacity, which refers to the maximum population size of a species that can be sustained in a specific environment, given constraints on available resources like food, water and habitat.

However, there are two problems with the logistic model:

1. The model cannot account for the effect of the recovery project as the population size is constantly decreasing;
2. The model is less applicable for imperiled plants with small populations.

To improve our model, we incorporate Allee effect which occurs in small or sparse population. Allee effect describes a positive relationship between individual fitness and population density[15]. In the case of imperiled plants, the larger the group population, the more likely an imperiled plant will survive extinction. We assume strong Allee effect for most 48 imperiled plants. Strong Allee effect can induce a critical population density below which the population growth rate is negative and extinction is likely to occur[16]. Thus, to protect an imperiled plant species from extinction, we only need to raise its population size above the critical population density to ensure positive population growth rate. This is aligned with our research that the goal of recovering endangered species is to increase their population to be self-sustaining. Our modified logistic equation is:

$$\frac{d\rho_i}{dt} = -r\rho_i\left(1-\rho_i\right)\left(\rho_i-\rho_0\right)$$

where ρ_0 represents the critical population density. Setting $r = 0.3$, $\rho_0 = 0.5$ and the maximum population density to be one unit, we plot a graph for two models:

[2] A planned fire applied to meet management objectives.

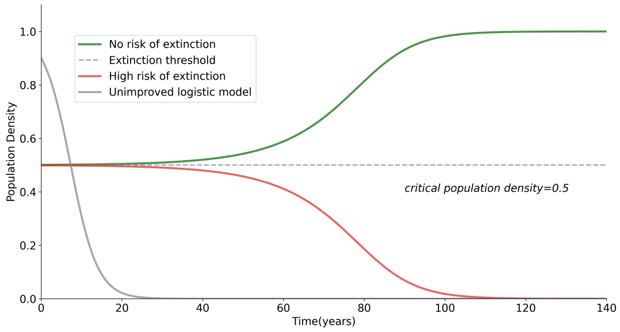

Figure 1: A comparison between unimproved logistic model (gray) and improved logistic model (red and green)

3.3.2 Feasibility Decay

According to our modified logistic model, population density of 48 imperiled species decreases with time if no actions are taken to protect them. To better protect imperiled species, actions should be taken as early as possible to ensure higher *feasibility of success*. However, it is impossible to start all projects simultaneously in the first year due to limited funding. Therefore, a discussion about how feasibility changes with time is important for us to better schedule the protection measures and come up with a long-term and reliable solution. In our model, the feasibility of a project will stop decreasing once the project starts, as recovery efforts are made to avoid extinction.

We assume that feasibility of a project at a particular year is directly proportional to the population density. This is justified since $f = 0$ when $\rho_i = 0$ (i.e. extinction) and $f = 1$ when $\rho_i = \rho_0$ (i.e. self-sustaining, no protection measure is needed).

$$f_i(t) \propto \rho_i(t)$$

Thus, we solve the differential equation using Mathematica to get the feasibility with time function of a project with initial feasibility slightly below 1 (in this case $f(0) = 0.998$):

$$f(t) = 1 - \frac{\sqrt{e^{rt} + 250000 \times e^{0.5rt}}}{e^{0.5rt} + 250000}$$

Since feasibility is the only variable in the differential equation, the function is invariant under translation along the horizontal axis. Therefore, all feasibility decay functions can be obtained by translating the previous function along the horizontal axis.

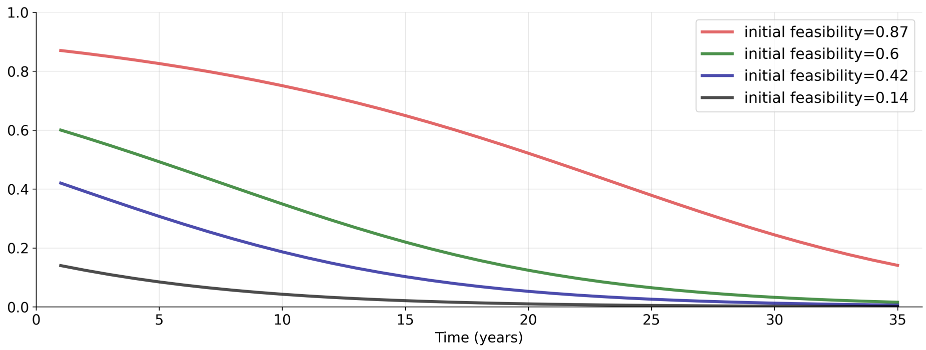

Figure 2: Feasibility decay for different projects when $r = 0.3$

In this model, the speed of decay can be adjusted by changing the extinction rate coefficient, r, of the species. In order make the effect of feasibility decay significant in our model, we set r to 0.3.

3.3.3 Priority Index

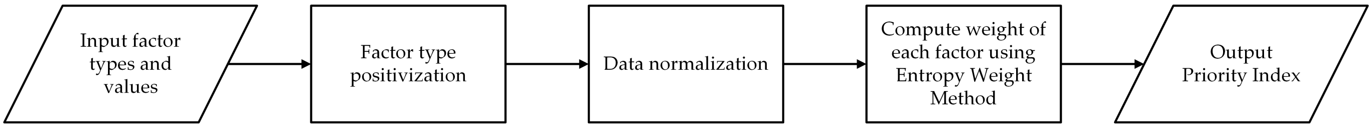

Figure 3: Flowchart of Priority Index computation

To evaluate which project the FRPCE board should prioritize, we use the Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) to construct the Priority Index, and the Entropy Weight Method to determine the weight of each factor. As *feasibility* decreases with time, the Priority Index will also vary with respect to time. In this section, we will compute the Priority Index of each project in each year using the five factors mentioned in the previous part: *benefit*, *taxonomic uniqueness*, *feasibility of success*, *total cost* and *total number of years*.

Firstly, we generate an $n \times m$ evaluation matrix $X(t)$ of year t, where n is the number of projects, m is the number of factors, and x_{ij} is the value of the jth factor of the ith project.

$$X(t) = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1m} \\ x_{21} & x_{22} & \cdots & x_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ x_{n1} & x_{n2} & \cdots & x_{nm} \end{bmatrix}$$

Next, we have to convert the factors into the same type. In this problem, *benefit* and *taxonomic uniqueness* are benefit factors[17], where greater values indicate greater priority. *Total cost* and *total number of years* are cost factors, where smaller values indicate greater priority. For *feasibility of success*, based on our decay function, it decreases the fastest at intermediate values and the slowest at extreme values. In our model, we want to prevent the *feasibility* from experiencing a rapid decay. Hence, we will prioritize projects whose current *feasibility* sits within an intermediate interval $[lb, ub]$ and we call *feasibility of success* an interval factor.

To unify the factor type, we perform **positivization** to convert all factors into benefit factors and transform matrix $X(t)$ into $\tilde{X}(t)$, the positivized evaluation matrix.

For benefit factors, there is no need for factor type conversion, i.e. $\tilde{x}_{ij} = x_{ij}$.
For cost factors,

$$\tilde{x}_{ij} = \max\{x_{ij}\} - x_{ij}, i = 1, ..., n$$

For interval factors, we denote the maximum distance of the data to the interval as M_j, i.e.

$$M_j = \max\{lb - \min\{x_{ij}\}, \max\{x_{ij}\} - ub\}, i = 1, ..., n$$

Performing positivization,

$$\tilde{x}_{ij} = \begin{cases} 1 - \frac{lb - x_{ij}}{M_j}, \ x_{ij} < lb \\ 1, \ lb \leq x_{ij} \leq ub \\ 1 - \frac{ub - x_{ij}}{M_j}, \ x_{ij} > ub \end{cases}, i = 1, ..., n$$

Then, we perform **normalization** to transform the evaluation matrix from $\tilde{X}(t)$ to $Z(t)$ to ensure all factors are on the same scale. We denote the normalized factor value as z_{ij}.

$$z_{ij} = \frac{\tilde{x}_{ij}}{\sqrt{\sum_{i=1}^{n} \tilde{x}_{ij}^2}},\ i = 1, ..., n,\ j = 1, ..., m$$

To determine the weight of each factor, subjective methods such as Analytic Hierarchy Process (AHP) are often used. However, the success of these methods depends on accurate judgement of factors' relative importance[18]. This is hard to achieve as humans are often obscured by their emotions and prejudices. On the other hand, objective weighting methods are able to reflect the amount of useful information in the set of factors given and thus are better at preventing human prejudices[19]. Hence, we choose to use the **Entropy Weight Method**. In our model, we assume that the weight of each factor is fixed (i.e. does not vary with time). We will use the data from the starting year (i.e. $Z(1)$) to compute the weights.

Step 1: the standardized value of the jth factor of the ith project is given by $p_{ij} = z_{ij}/\sum_{i=1}^{n} z_{ij}$

Step 2: the entropy value of the jth factor is defined as $e_j = -\sum_{i=1}^{n} p_{ij} \ln(p_{ij})/\ln n$ [20]

Step 3: calculate the weight of the jth factor $w_j = (1 - e_j)/\sum_{j=1}^{m}(1 - e_j)$

From the data given, we obtain w_i ($i = 1 - 5$ represents the weight of benefit, taxonomic uniqueness, feasibility of success, total cost and total number of years respectively):

w_1	w_2	w_3	w_4	w_5
0.141	0.062	0.486	0.095	0.217

Table 2: Weight of each factor given by Entropy Weight Method

Now we proceed to calculate the Priority Index of the ith project in a particular year. Firstly we define Z_j^+ and Z_j^- as the maximum and minimum value in the jth column of matrix Z.

$$Z^+ = \left(Z_1^+, ..., Z_m^+\right) = \left(\max\left\{z_{11}, ..., z_{n1}\right\}, ..., \max\left\{z_{1m}, ..., z_{nm}\right\}\right)$$

$$Z^- = \left(Z_1^-, ..., Z_m^-\right) = \left(\min\left\{z_{11}, ..., z_{n1}\right\}, ..., \min\left\{z_{1m}, ..., z_{nm}\right\}\right)$$

Then, we define D_i^+ and D_i^- as the weighted Euclidean distance from the ith project to the best possible project and the worst possible project respectively.

$$D_i^+ = \sqrt{\sum_{j=1}^{m} w_j\left(Z_j^+ - z_{ij}\right)^2},\ D_i^- = \sqrt{\sum_{j=1}^{m} w_j\left(Z_j^- - z_{ij}\right)^2}$$

Therefore, the Priority Index of the ith project in year t after normalization, $PRI_i(t)$, can be computed as follows:

$$PRI_i(t) = \frac{S_i}{\sum_{i=1}^{n} S_i}$$

where $S_i = \frac{D_i^-}{D_i^+ + D_i^-}$.

The Priority Index we obtained can be found in **Appendix**. Due to space limit, we only display the Priority Index of every 5 years.

3.4 Crafting Fundraising Schedule

Having obtained the Priority Index $PRI_i(t)$ of each project in each year, we can now use Dynamic Programming with Greedy Algorithm to determine a fairly efficient schedule that demonstrates which projects should be prioritized to invest in each year amongst our 48 recovery projects.

3.4.1 Dynamic Programming

Firstly, we use Dynamic Programming (DP) to decide which projects to fund in each individual year. For now, we assume none of the projects has been selected.

Analogous to 0-1 Knapsack Problem, our goal is to maximize the total Priority Index $PRI_i(t)$ of year t, while keeping the total funding that selected projects require in each year under a reasonable limit, $MaxFund$. In our case, each of the 48 projects has its value in terms of Priority Index $PRI_i(t)$, and its costs in the first year, $Cost_i$ (given in the database). The total costs of selected projects cannot exceed $MaxFund$. Each project is either chosen to be funded, or not selected in that year.

We denote the state function $DP(i, Fund)$ as the maximum $\sum PRI_i(t)$ obtainable when we are to choose among the first i ($i \leq 48$) projects, under the constraint that total yearly costs required do not exceed $Fund$ ($Fund \leq MaxFund$). Hence, for a particular i and $Fund$, we have two options:

1. If the ith project is not selected, the maximum Priority Index obtainable in $DP(i, Fund)$ is equivalent to $DP(i-1, Fund)$.
2. If the ith project is selected, we can gain additional PRI_i, on top of what can be obtained from $DP(i-1, Fund - Cost_i)$.

We should select the option that gives a higher total Priority Index obtainable. Hence, the recurrent equation can be written as:

$$DP(i, Fund) = \mathbf{max}\,\{DP(i-1, Fund), DP(i-1, Fund - Cost_i) + PRI_i(t)\}$$

To visualize this concept, we design the following pseudocode. Note that in coding, function $DP(i, Fund)$ is represented by a 2-D array of i rows and $Fund$ columns, so $Fund$ must be an integer. Hence, during DP, we rounded up the costs of each project to integer. Rounding up also makes sure that total costs from selected projects will not exceed $MaxFund$.

Algorithm 1 Dynamic Programming

```
read MaxFund, Cost[1...48], PRI[1...48]
declare DP: array [1...48, 1...MaxFund] of float
for i ← 1 to 48 do
    Fund ← MaxFund
    repeat
        DP[i][Fund] = max(DP[i − 1][Fund], DP[i − 1][Fund − Cost[i]] + PRI[i])
        Fund ← Fund − 1
    until Fund ≤ Cost[i]
    repeat
        DP[i][Fund] ← DP[i − 1][Fund]
        Fund ← Fund − 1
    until Fund ≤ 0
end for
```

Now that we have used $DP(i, Fund)$ as memoization of our results, we can backtrack the projects that were selected from $DP(48, MaxFund)$, which stores the highest Priority Index. The pseudocode is shown below.

Algorithm 1 Dynamic Programming - Backtrack

```
declare ProjectStatus: array [1...48] of boolean
BackTrack(48, MaxFund)
procedure BackTrack(i, w)
    if i = 0 then                                   ▷ Base case
        return
    else if DP[i][w] > DP[i − 1][w] then
        ProjectStatus[i] ← 1                        ▷ Project i is selected
        BackTrack(i − 1, w − Cost[i])
    else
        ProjectStatus[i] ← 0                        ▷ Project i is not selected
        BackTrack(i − 1, w)
    end if
end procedure
```

3.4.2 Greedy Algorithm

As our schedule spans across N years, the main idea of Greedy Solution is that we use DP to select the optimal set of projects to fund in each year. By iterating this procedure N times, the total Priority Index $\sum PRI_i$ in each of the N years will also be maximized.

Here we explain the structure of our Greedy Algorithm:

Step 1: Before the start of year 1, we set a $FundCap$ that restricts the maximum cost required in each year. We initialize the $Cost$ and PRI for each project.

Step 2: At the start of each iteration, we check for all projects that are already selected from previous years and are currently ongoing. As these projects have to continue to be funded this year (Model Objective 3), we deduct their costs from $FundCap$

and assign the remaining costs to $MaxFund$, which is the money limited for DP to choose among unselected projects.

$$MaxFund = FundCap - \sum Costs\ of\ ongoing\ projects$$

Step 3: At the start of each iteration, we also refresh the $PRI_i(t)$ for each unselected projects, based on the Feasibility Decay function $f_i(t)$ (See section 3.3.2), where t here refers to the corresponding year.

Step 4: Perform Dynamic Programming among unselected projects to choose the combination that gives the highest $\sum PRI_i(t)$ of that year, using the updated values of $MaxFund$ and $PRI_i(t)$.

Step 5: Update the schedule for newly selected projects. Increase the year by 1.

Step 6: Repeat **Step 2 to Step 5** until N iterations are completed. Display the schedule and evaluate its effectiveness (See Section 3.4.3).

However, we notice that if we set the total number of years N to be constant, after N iterations, some projects which have been chosen are not completed because they started too late. To be more flexible, we may adjust N at the end of Greedy Algorithm, by examining those ongoing yet incomplete projects:

1. If a project requires only a few years left to complete and has relatively high Priority Index among the incomplete projects, we may extend the total number of years N until this project is complete.
2. Otherwise, we may abandon this project entirely, remove its costs from our schedule, and repeat Greedy Algorithm to find the best schedule among the remaining projects. Total number of years N will become the end year of the last completed project.

This small modification may increase the number of projects recovered, while saving costs from projects that are unrealistic to complete.

Here, we present the complete flowchart that illustrates the above process:

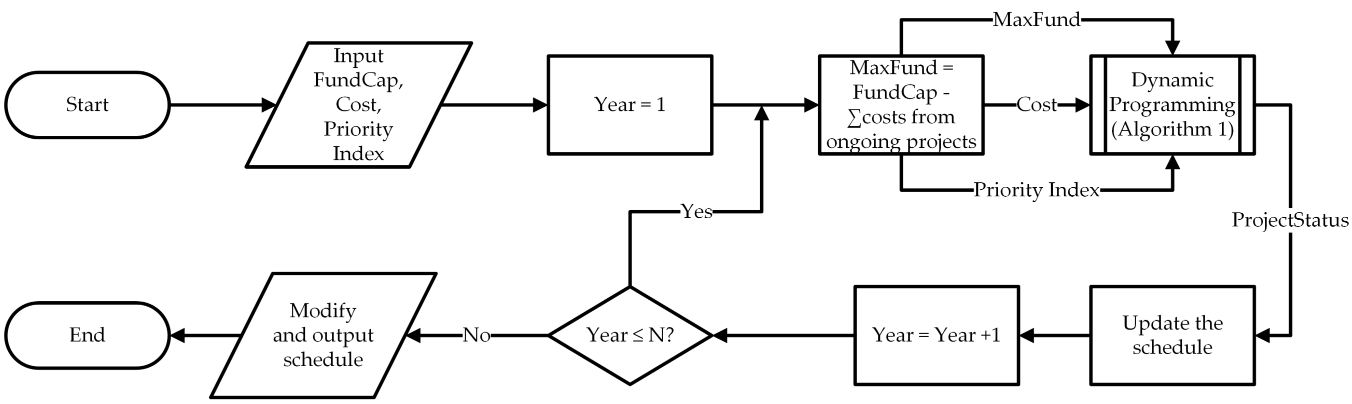

Figure 4: Greedy Algorithm to determine initial schedule

There are several reasons why we can apply Greedy Solution here:

1. We want to achieve an overall high Priority Index since $PRI_i(t)$ takes the effect of feasibility decay $f_i(t)$ into account. By maximizing sum of Priority Index, we can conserve plant species that need urgent actions first.

2. We also observe that in the database provided, the funding required for most projects decreases over time (*). Hence, this ensures that the total costs from ongoing projects in subsequent years will not exceed $FundCap$, and hence there will be remaining $MaxFund$ for DP in the subsequent years.

(*) We notice that in the data given, the budget for 47 out of 48 projects decreases continually over the years, except for 1-Flowering Plants-486 whose budget falls to zero from year 4 but suddenly spikes in year 10. Here, we temporarily modify the budget plan of this project to be in descending order throughout the 10 years, by transferring its year 10 costs to the previous years. Hence, in the preceding years the project requires more funds than what it actually requires, so we treat these extra funds as savings for the 10th year.

3.4.3 Evaluating Our Initial Schedule

Using the above algorithms, we generate an initial schedule, setting the total number of years $N = 35$ and annual funding cap $FundCap = \$500,000$.

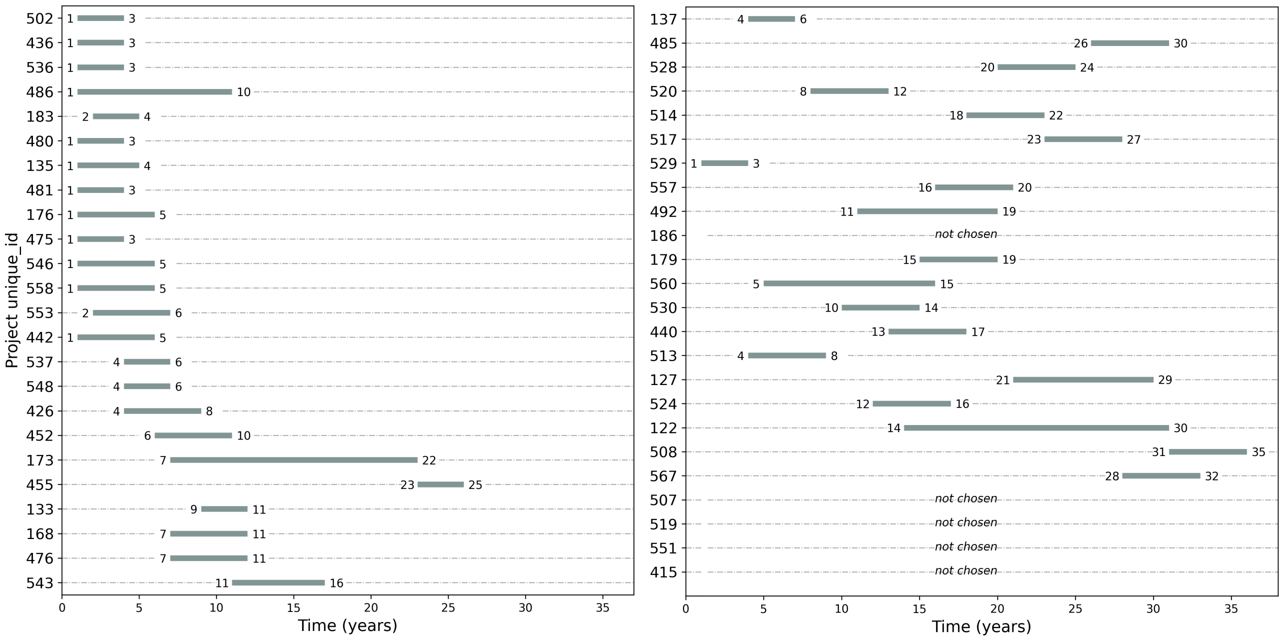

Figure 5: Our proposed schedule obtained by DP and Greedy Algorithm

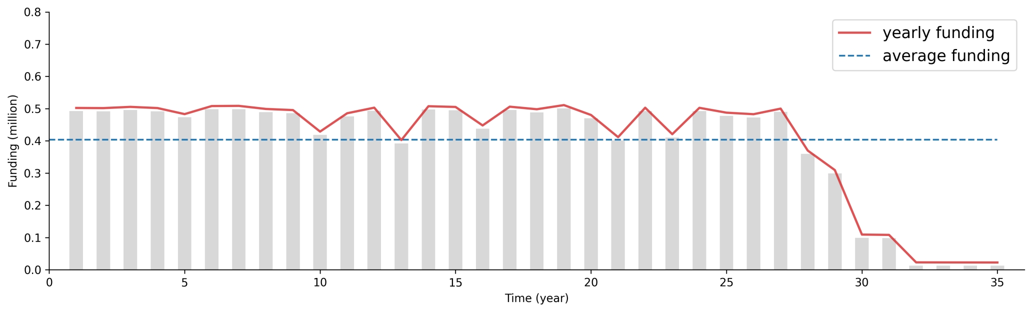

Figure 6: Yearly funding required

To evaluate the effectiveness of a schedule, we first calculate its **expected net benefit**, EB, using the formula below[21]:

$$EB = \sum_{i \in S} \frac{(w_1' b_i + w_2' u_i) f_i}{c_i}$$

where S is the set of selected projects, $w_1' = \frac{w_1}{w_1+w_2}$ and $w_2' = \frac{w_2}{w_1+w_2}$ in which w_1 and w_2 are the weights assigns to *benefit* b_i and *taxonomic uniqueness* u_i in Entropy Weight Method respectively. f_i is the *feasibility* of the ith project when it starts, and c_i is the *total cost* of the ith project. EB takes into account all factors except for the *total number of years* of projects since our schedule will span a significantly longer period compared to the duration of individual projects. Hence duration of each project is not very relevant to the benefits gained from its completion.

It is noteworthy that we use PRI_i to determine the priority order of project selection but not EB. This is because EB does not include the effect of feasibility decay and the total number of years taken to finish each project. If we use EB to determine the priority order of each project, we are more likely to optimize yearly benefit instead of long-term benefit.

Moreover, EB alone cannot comprehensively evaluate our initial schedule since it does not reflect the fluctuation of yearly funding requirement. In order to take fluctuation into account, we use **total score** TS,

$$TS = \frac{1000 \times EB}{\sigma}$$

where σ represents the **standard deviation** of yearly funding and 1000 is a scaling factor.

From our initial schedule, we obtain $EB = 81.88$, $\sigma = 109443$ and $TS = 0.748$. While our initial schedule can provide a reasonably good EB, it has a large fluctuation towards the end of the schedule, leading to a poor TS. To maximize TS, we present an optimization algorithm in the next section.

3.5 Schedule Optimization

From the previous section, we can obtain a schedule using DP and the Greedy Algorithm. However, there are two limitations to the schedule. Firstly, it fails to account for our objective to minimize the fluctuations of yearly funding. As shown in Figure 6, yearly funding falls sharply from year 27, which leads to a high σ and thus a low TS. Secondly, the use of greedy algorithm only ensures an optimal solution at each iteration. The schedule obtained may not be a global optimum. Therefore, to optimize our schedule, we use the Genetic Algorithm (GA).

3.5.1 Genetic Algorithm (GA)

First proposed by J. H. Holland[22] and L. J. Fogel et al.[23] based on Darwin's Evolutionary Theory, GA starts from an initial population that represents admissible solutions to a problem through suitable coding. Leveraging on the principle of variation by crossover, mutation and selection of the fittest, it iterates through generations and solves the problem satisfactorily[24].

GA has multiple advantages. Firstly, it is able to deal with complex optimization problems with different types of objective (fitness) function, be it stationary or non-stationary (changes with time), linear or nonlinear, discrete or continuous. Secondly, it supports multi-objective optimization. Thirdly, since individuals in the population act like independent agents, they can explore the search space in many directions simultaneously, which significantly reduces the possibility of being stuck in a local optimum[25].

To implement GA, we abstract our schedule k into a chromosome. Each of the 48 projects represents a distinct gene and the starting year of the ith project, $T_k^{(i)}$, is the "genetic information" that will be modified. Here are the specific notations we will use for GA:

Symbol	Meaning
P_c	Probability of crossover
P_m	Probability of mutation
a	Maximal number changed to the starting year if mutated
q	Number of population (children produced) in each generation
s	Number of children selected in each generation
M	Total number of generations

In the following part we will use 2 parents and 2 children to illustrate the algorithm. Firstly, after initializing the population, we apply **2-point crossover** to the parents. To do this, we randomly select two positions on the chromosome, dividing the chromosome of each parent into three parts. Next, we switch the middle parts at probability P_c to obtain two children.

Then, at probability P_m, each gene in a child would **mutate** by adding a random integer ranging from $-a$ to a. There are some restrictions to mutation. Firstly, the starting and ending year of each project after mutation must still be within $[1, N]$. Secondly, while aiming to reduce fluctuation, a mutated solution must still possess at least $95\% EB$ of our initial solution to prevent excessive decline in EB when certain projects are delayed. The process of crossover and mutation is illustrated in Figure 7.

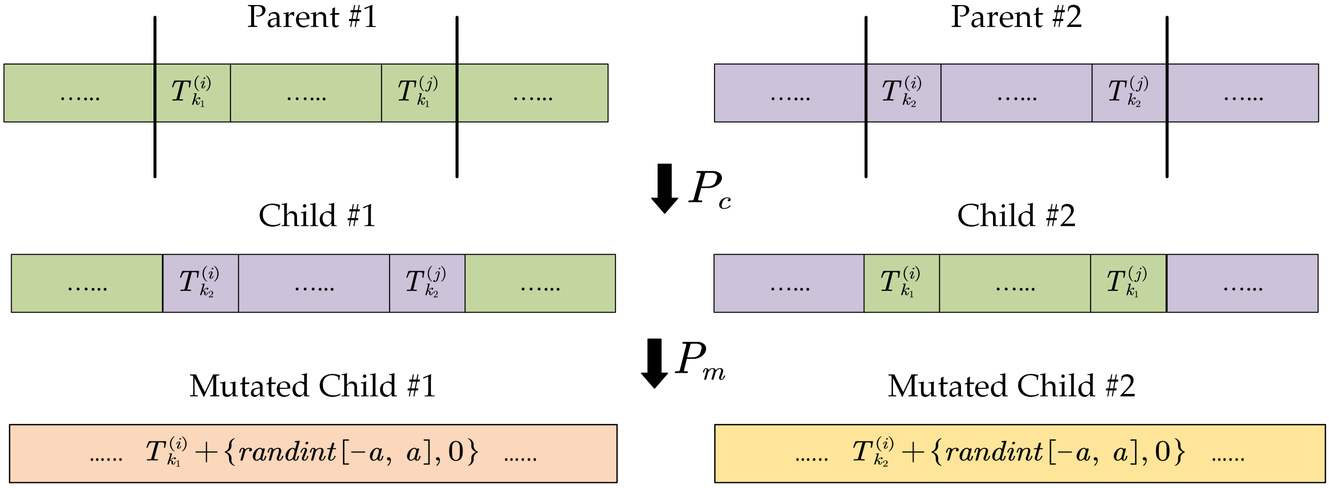

Figure 7: Crossover and mutation

Next, according to the principle of "survival of the fittest", we apply **tournament selection** to select 2 out of the q children as the parents of the next generation. We carry out tournament selection twice: each time we randomly select $\frac{3}{4}$ of the population q, and pick the individual with the highest TS (fitness level) to be one of the 2 parents next year. Note that we do not directly select the best 2 individuals out of the whole population, but allow for some "noise" – or probability – of selecting the "sub-best" solutions, so that our generations will be more diversified and will not be stuck in local optimum solution. After the 2 parents are selected, they will crossover and mutate again to produce the next generation of solutions. The whole process will iterate through M generations and we will keep track of the overall best solution.

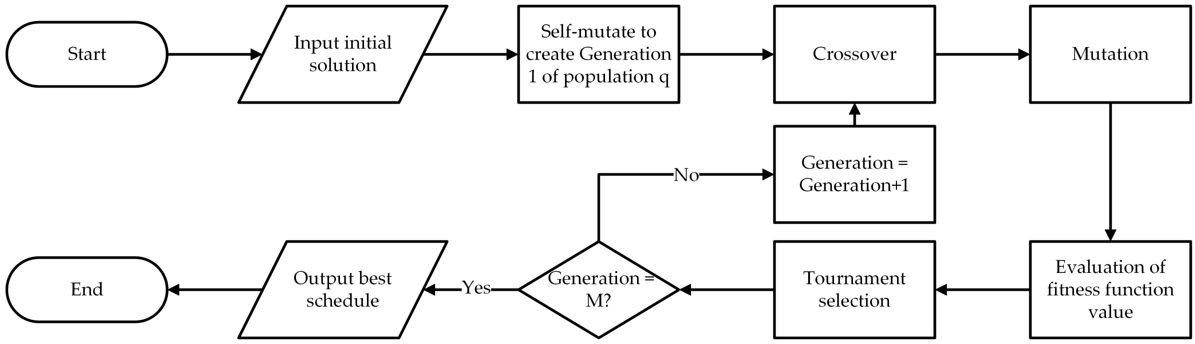

Figure 8: Flowchart of Generic Algorithm

In our problem, we take $a = 10$, $q = 100$, $s = 2$, $M = 300$. As the initial solution only consists of one individual (Figure 5, initial schedule obtained from DP), it will self mutate 100 separate times to produce the first generation and the crossover process will only start from the second generation. We use tournament selection to select 2 children in each generation as parents of the next generation which will crossover and mutate to produce children for the next generation. We obtain the following optimized schedule and we compare it with the initial schedule:

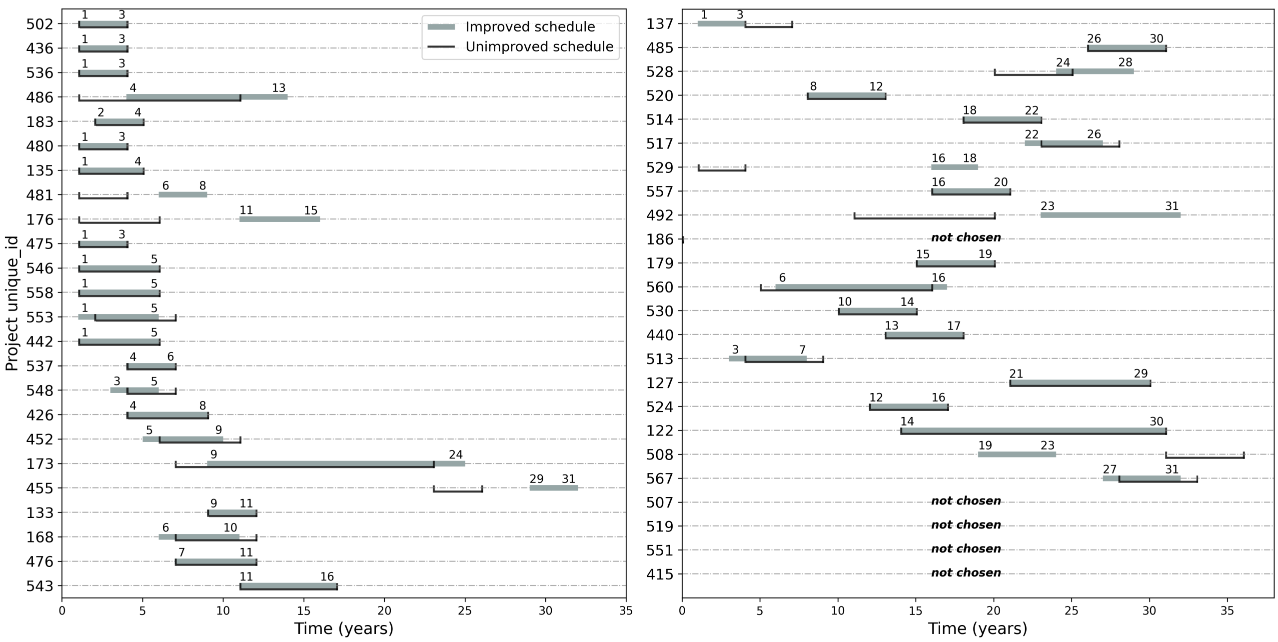

Figure 9: Our improved schedule obtained by GA

3.5.2 Result Comparison

We plot the yearly funding of two schedules (Figure 10), and the improved one has two advantages: firstly, the unimproved schedule has a sharp decline from year 28 whereas our improved one achieves a relatively consistent funding throughout the duration, leading to a much lower fluctuation; secondly, the improved schedule takes 3 years less to finish the same number of projects without exceeding the fund limit, indicating that the fund is used more efficiently.

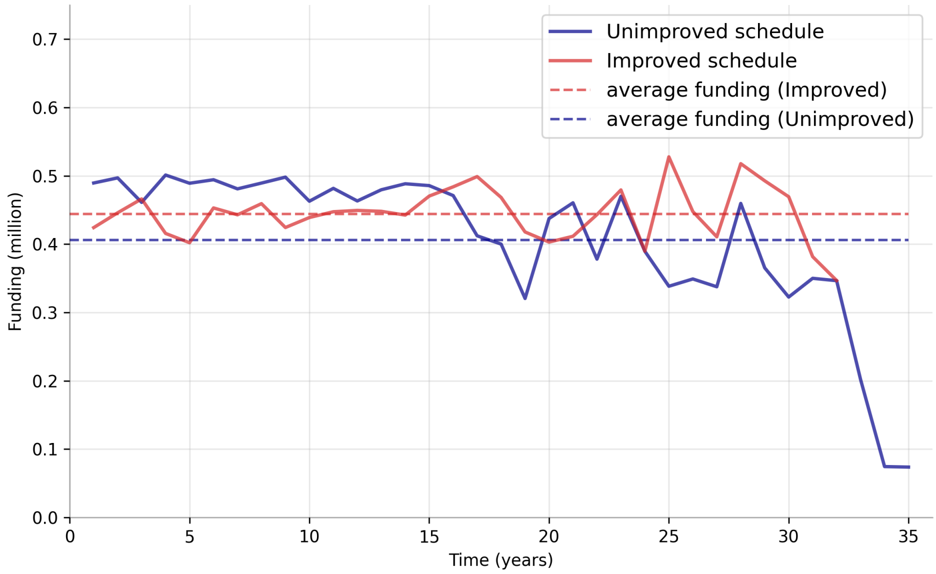

Figure 10: Comparison of two schedules

Furthermore, another advantage of using GA is that we are able to deal with data of different trends. In the data given in the question, funding requirement for 47 of 48 projects decreases continually as time goes on. In reality, yearly cost of a project can go up and down as time goes on. With GA, we can always minimize the yearly funding fluctuation by rearranging the order of projects. Here we give an example where the funding required for each project is reversed (i.e. the funding required for projects in each year increases with time). As shown in Figure 11, the unimproved schedule has a huge spike in funding during the starting years and fluctuates more violently than the improved schedule. This suggests significant improvements using GA.

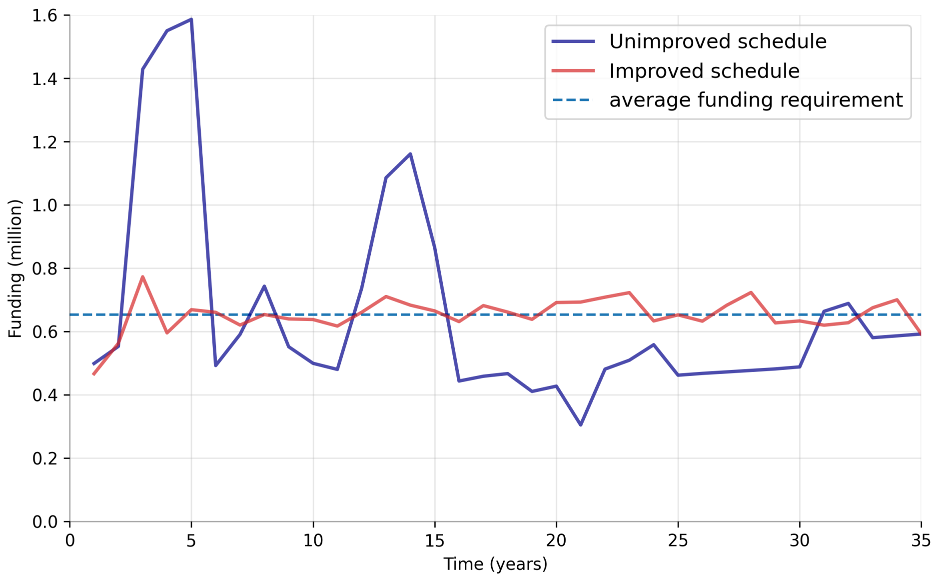

Figure 11: Reversed data

3.5.3 Different Initial Conditions

For GA, different initial conditions may lead to different optimized outcomes. If the initial solution is far off from the global maximum, solution is likely to converge to a local maximum instead. In our model, we choose the result obtained from greedy algorithm and DP as our initial condition. To show why this initial schedule is necessary, we further obtained the result of GA if the initial condition is set randomly.

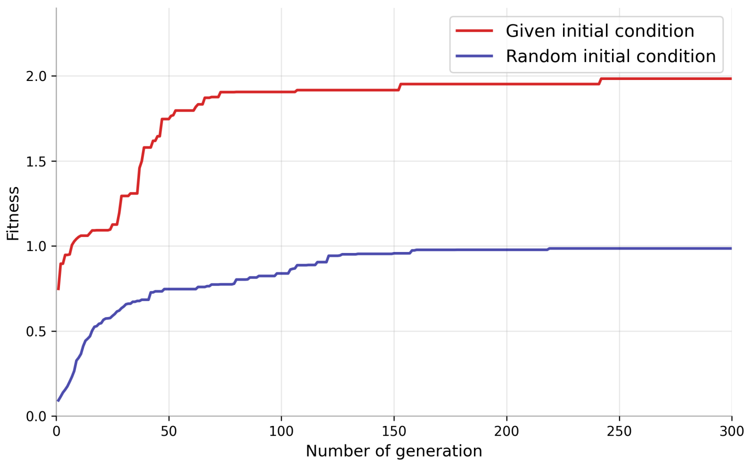

Figure 12: Convergence plot

In Figure 12, the random initial condition has a lower fitness value. Besides, it does not converge to the optimized fitness value obtained by the one where the initial condition is given by DP. One possible reason may be the undetermined pool of projects chosen in the first year. In order to minimize the yearly funding fluctuation, GA may choose those projects which meet the funding requirement but are of low benefit values. This ultimately restrains the optimized fitness value.

4 Results

From DP, we obtained the relationship between the number of completed projects and $FundCap$.

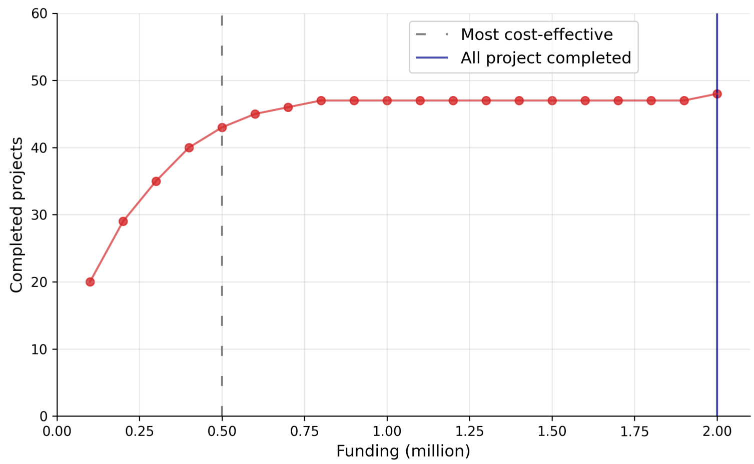

Figure 13: Relationship between the number of completed projects and yearly funding cap

For the FRPCE Board to achieve a "long-term and reliable funding" where omission of certain projects is allowed, we recommend **$500,000** as the minimum fundraising required since it can complete a relatively high number of projects (43 of 48) while keeping the cost low. On the other hand, completing all projects within a reasonable time period (40 years) requires a maximum yearly funding cap of at least **$2,000,000**. We also recommend this schedule to the Board if they insist on completing all projects.

Subsequently, we use GA to optimize the initial schedule generated by DP and Greedy Algorithm. The priority order of projects in these two schedules are presented in Figure 14. The value in each cell represents the starting year of the project and NC means the project is not chosen.

$FundCap$	EB	σ	TS
$\$500,000$	77.801	39215	1.984
$\$2,000,000$	85.744	126002	0.681
$FundCap$	$\lvert S\rvert$	Duration	<yearly cost>
$\$500,000$	43	32	$\$444,437.25$
$\$2,000,000$	48	40	$\$1,274,652.71$

Table 3 Results of the recommended schedules

(S is the set of selected projects and $|S|$ is number of completed projects; <yearly cost> is the annual yearly cost.)

Yearly cap / Project	$500,000.00	$2,000,000.00
Plants-502	1	1
Plants-436	1	1
Plants-536	2	1
Plants-486	1	1
Plants-183	30	1
Plants-480	2	1
Plants-135	2	1
Plants-481	1	1
Plants-176	4	1
Plants-475	1	2
Plants-546	1	3
Plants-558	1	2
Plants-553	1	1
Plants-442	1	9
Plants-537	21	1
Plants-548	13	1
Plants-426	5	1
Plants-452	6	5
Plants-173	13	1
Plants-455	30	18
Plants-133	9	1
Plants-168	6	1
Plants-476	7	1
Plants-543	11	1
Plants-137	4	15
Plants-485	17	4
Plants-528	20	4
Plants-520	8	8
Plants-514	15	2
Plants-517	25	16
Plants-529	1	1
Plants-557	18	14
Plants-492	11	1
Plants-186	NC	1
Plants-179	22	4
Plants-560	4	6
Plants-530	10	8
Plants-440	14	5
Plants-513	3	14
Plants-127	23	4
Plants-524	12	5
Plants-122	16	4
Plants-508	25	13
Lichens-567	28	10
Plants-507	NC	17
Plants-519	NC	11
Plants-551	NC	7
Plants-415	NC	21

Figure 14: Color-coded priority order of recommended schedules

5 Sensitivity Analysis

To test the sensitivity of our model, we use the schedule with yearly funding capped at $\$500,000$ and increase the extinction rate coefficient r by 1% (0.003) and 5% (0.015) respectively. We observe that the initial schedule obtained after DP and Greedy Algorithm does not change when r varies. This attests to the robustness of our DP and Greedy solution.

However, one concern is that as the population of the species decreases faster, feasibility will decrease faster, resulting in a lower total expected net benefit that can be potentially yielded. Hence, we will measure the change in EB, TS and yearly funding.

r	EB	EB % change	TS	TS % change
0.3	77.8	-	1.98	-
0.303 (+1%)	77.4	-0.46%	1.93	-2.72%
0.315 (+5%)	75.9	-2.42%	1.85	-6.75%

Table 4: Change in EB and TS

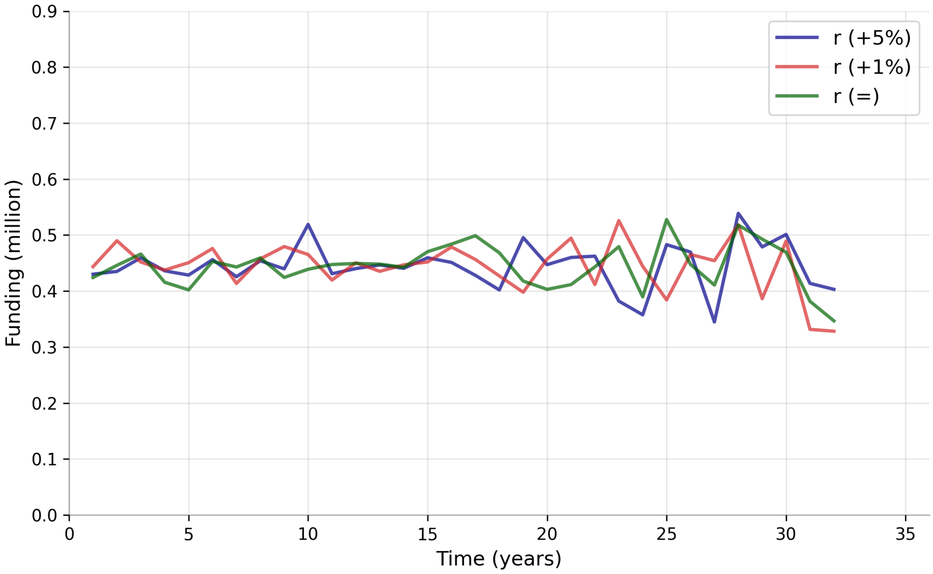

Figure 15: Change in annual funding when r is adjusted

From Table 4, we can see that when r is set to increase, the decrease in both EB and TS falls within a reasonable range. Moreover, from Figure 15, our final yearly cost does not change significantly when we adjust r. This shows that our optimization methods are generally stable such that the change in r will not excessively affect the optimal schedule found.

6 Strengths and Weaknesses

6.1 Strengths

- Our model is realistic since we recognized that imperiled plant species without conservation will continue to decline in population size. Because of this increasing risk of extinction, the possibility of a conservation project saving the species from extinction will definitely decrease, which means the feasibility of the project will decrease.

- We used two algorithms to generate the initial schedule and optimize it respectively. By doing so, we can obtain an optimal solution with both a relatively high total benefit and a minimized fluctuation in yearly expenditure.

- Our model can keep annual funding relatively constant even if the projects' yearly costs do not follow a decreasing trend. The use of Genetic Algorithm ensures that our initial schedule is optimized and the yearly budget will not be exceeded by an unreasonable amount.

- Our model is highly customizable since the FRPCE Board could enter their perceived relative importance of each factor in Priority Index.

- We used impartial and objective methods to determine the Priority Index. TOPSIS can reflect the underlying difference between projects. The Entropy Weight Method ensures the objectivity in decision-making by computing weight based on the amount of information that can be derived from the data.

6.2 Weaknesses

- Feasibility decay function is idealized and may not describe the real life situation since the population of endangered plant species are influenced by many factors like environmental changes, individual species characteristics and other contingencies that cannot be easily quantified.

- Projects with very high feasibility are treated the same as projects with very low feasibility when computing Priority Index. This may cause projects with low feasibility being chosen instead of those with high feasibility. This is not preferable for maximizing our expected net benefit.

- We did not eliminate projects when their feasibility decay close to 0. Other factors like a high benefit score contribute to its Priority Index, which may lead to it being chosen, despite the fact that the project cannot contribute to total expected net benefit.

7 Conclusion

In conclusion, we managed to address the problem requirement fully by identifying objectives and factors involved in the Board's decision-making, and generating optimal schedules for different funding caps. We successfully employed DP to select the most urgent and valuable projects and generated an initial schedule. Afterwards, we used Genetic Algorithm to optimize our schedule, in particular minimized fluctuations in yearly expenditure. From our model, we recommended the schedule obtained using $\$500,000$ as the yearly cap to the Board since it exhibits high cost-effectiveness. The Board could also use $\$2,000,000$ as the yearly cap if they hope to complete all 48 recovery projects.

In the future, the limitations of our feasibility model could be addressed by using data gathered by FWS to fit the feasibility decay curve.

While our model does provide a suggested priority order of funding for the 48 imperiled plant species, a realistic concern would be how to adapt to a scenario where annual budgets are changing and projecting future available funding based on given revenue in recent years. Nevertheless, our model is still holistic in solving the resource allocation problem and can be easily expanded to suit more complicated contexts.

8 Memo

Date: 17/11/2020

To: The Florida Rare Plant Conservation Endowment (FRPCE) Board

From: HiMCM Team #10997

Subject: Recommendation of funding schedule for 48 imperiled plant species conservation

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As a group of students passionate about mathematical modeling, we recently learnt about the Board's consistent efforts in funding imperiled plant species conservation projects in Florida. We are also aware of the difficulty you are facing in prioritizing the various recovery projects. Given the data set of 48 species' conservation projects, we utilized mathematical models and computer algorithms to generate funding schedules that can maximize conservation benefits, and at the same time, minimize fluctuations in yearly expenditures.

In quantifying an aggregate expected net benefit score, we take into account the feasibility of success, benefit, taxonomic uniqueness, and total costs of each project. A good funding plan should achieve a high project success rate, save species with the most conservation value. We also recognize that the Endowment would not like to spend too much or too little money on a specific year, wasting or straining its yearly budget. Hence, our proposed plan is optimized to have relatively stable yearly funding.

Based on our algorithm, we suggest $500,000 as the annual funding cap since 43 out of 48 projects can be completed and the schedule is the most cost-effective. Meanwhile, we present $2,000,000 as the minimum yearly funding that could finish funding all projects in a reasonable time span. The priority order of funding we obtained are listed in Figure 14. Under the specific annual funding column, the number corresponding to a project indicates the start year of the project. For instance, 1-Flowering Plants-135 should be funded starting from Year 2 when the Board adopts the $500,000 funding cap. When the value is NC, this conservation project is not chosen, usually because the yearly cost greatly exceeds the annual budget or the expected net benefit is too low.

You can understand the rationale behind the schedule easily. In our model, we first used a Priority Index to decide which projects to be funded first. Projects with greater benefit and uniqueness are prioritized since they yield more conservation value. We also prioritize shorter projects and those which require less funding to accommodate for more projects in the future. Feasibility of a project was modeled to be decreasing logistically with time, which means the feasibility of a project decreases the fastest near the median value. To achieve a high overall evaluation score, we prioritize these projects since an extremely high/low feasibility does not change appreciably with time. Based on our results, projects like 1-Flowering-Plants-502/481 are done during the first year as they have low total cost and short time span, fair benefit and uniqueness as well as intermediate feasibility values. Projects with very high feasibility like 514/179 are funded later since they are less urgent. Our model was tested to have the maximum expected net benefit and minimum fluctuations in yearly expenditures.

We hope that by selecting and implementing the appropriate funding schedule from our results table, your organization will be able to bring maximum welfare to Florida's rare plants and their habitat. An optimal funding schedule is crucial to conservation efforts in Florida given pressing extinction threats imposed by climate change, land use change and urban development. Species conservation still has a long way to go but we hope that our efforts can contribute to your decision-making process to save endangered species.

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Appendix (Priority Index)

unique_id	Year 1	Year 5	Year 10	Year 15	Year 20	Year 25	Year 30	Year 35
Plants-502	0.024	0.025	0.028	0.028	0.023	0.019	0.018	0.017
Plants-436	0.022	0.018	0.015	0.014	0.014	0.015	0.015	0.016
Plants-536	0.018	0.021	0.027	0.035	0.044	0.055	0.058	0.042
Plants-486	0.023	0.024	0.026	0.031	0.033	0.026	0.020	0.016
Plants-183	0.016	0.014	0.013	0.013	0.014	0.015	0.015	0.016
Plants-480	0.022	0.018	0.015	0.014	0.014	0.015	0.015	0.016
Plants-135	0.024	0.025	0.021	0.018	0.016	0.016	0.016	0.016
Plants-481	0.024	0.025	0.028	0.033	0.035	0.028	0.022	0.019
Plants-176	0.024	0.024	0.019	0.016	0.015	0.015	0.015	0.016
Plants-475	0.024	0.020	0.016	0.015	0.015	0.015	0.016	0.016
Plants-546	0.024	0.025	0.025	0.020	0.017	0.016	0.016	0.016
Plants-558	0.023	0.025	0.027	0.033	0.039	0.032	0.025	0.020
Plants-553	0.024	0.025	0.027	0.031	0.026	0.021	0.018	0.017
Plants-442	0.024	0.025	0.025	0.020	0.017	0.016	0.016	0.016
Plants-537	0.023	0.025	0.028	0.033	0.038	0.031	0.024	0.020
Plants-548	0.017	0.020	0.025	0.031	0.040	0.050	0.051	0.037
Plants-426	0.023	0.020	0.016	0.014	0.014	0.014	0.014	0.015
Plants-452	0.026	0.027	0.028	0.023	0.019	0.018	0.017	0.017
Plants-173	0.014	0.012	0.010	0.010	0.010	0.011	0.011	0.011
Plants-455	0.024	0.020	0.017	0.015	0.015	0.016	0.016	0.017
Plants-133	0.022	0.023	0.025	0.026	0.022	0.019	0.018	0.017
Plants-168	0.024	0.025	0.027	0.027	0.022	0.018	0.017	0.016
Plants-476	0.024	0.025	0.027	0.032	0.029	0.023	0.019	0.017
Plants-543	0.022	0.023	0.026	0.024	0.020	0.017	0.016	0.015
Plants-137	0.012	0.013	0.014	0.016	0.022	0.035	0.052	0.073
Plants-485	0.024	0.020	0.016	0.015	0.015	0.016	0.016	0.017
Plants-528	0.026	0.027	0.029	0.026	0.021	0.019	0.018	0.018
Plants-520	0.023	0.025	0.027	0.033	0.037	0.029	0.023	0.019
Plants-514	0.013	0.013	0.014	0.014	0.016	0.019	0.026	0.038
Plants-517	0.024	0.019	0.016	0.015	0.015	0.016	0.016	0.017
Plants-529	0.022	0.023	0.024	0.022	0.018	0.017	0.016	0.016
Plants-557	0.024	0.024	0.019	0.016	0.015	0.015	0.015	0.016
Plants-492	0.023	0.020	0.015	0.013	0.013	0.013	0.013	0.014
Plants-186	0.015	0.012	0.010	0.009	0.009	0.009	0.010	0.010
Plants-179	0.013	0.013	0.014	0.016	0.022	0.036	0.056	0.077
Plants-560	0.015	0.018	0.024	0.031	0.039	0.050	0.053	0.040
Plants-530	0.019	0.022	0.027	0.032	0.041	0.050	0.041	0.029
Plants-440	0.023	0.023	0.019	0.016	0.015	0.014	0.015	0.015
Plants-513	0.021	0.019	0.015	0.014	0.013	0.014	0.014	0.015
Plants-127	0.023	0.024	0.024	0.019	0.016	0.015	0.014	0.014
Plants-524	0.023	0.024	0.025	0.022	0.018	0.016	0.015	0.015
Plants-122	0.009	0.009	0.009	0.010	0.011	0.015	0.023	0.036
Plants-508	0.015	0.013	0.012	0.012	0.013	0.014	0.014	0.015
Lichens-567	0.016	0.014	0.013	0.013	0.013	0.014	0.014	0.015
Plants-507	0.021	0.023	0.020	0.016	0.014	0.013	0.013	0.013
Plants-519	0.023	0.024	0.025	0.021	0.017	0.015	0.014	0.014
Plants-551	0.022	0.023	0.025	0.023	0.019	0.016	0.014	0.014
Plants-415	0.017	0.018	0.020	0.022	0.018	0.013	0.009	0.007